'Incompleteness': Waiting for Gödel

Einstein presents the first Albert Einstein Award for excellence in the natural sciences to Julian Schwinger and Kurt Gödel (first and second from right) at Princeton, 1951.

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INCOMPLETENESS
The Proof and Paradox of Kurt Gödel.
By Rebecca Goldstein. Illustrated. 296 pp. Atlas Books/ W. W. Norton & Company. $22.95.

READERS' OPINIONS

R EBECCA GOLDSTEIN, as anyone knows who has read her novels -- particularly "The Mind-Body Problem" -- understands that people are thinking beings, and the mind's loves matter at least as much as the heart's. After all, she's not just a novelist, but a philosophy professor. She casts "Incompleteness," her brief life of the logician Kurt Gödel (1906-78), as a touching intellectual love story. Though Gödel was married, his wife barely appears here; as Goldstein tells it, his romance was with mathematical Platonism, the idea that the glories of mathematics exist eternally beyond our grasp. Gödel's Platonism inspired him to deeds as daring as any knight's: he proved his famous incompleteness theorem for its sake. His Platonism also set him apart from his intellectual contemporaries. Only Einstein shared it, and could solace Gödel's loneliness, Goldstein argues. A biography with two focuses -- a man and an idea -- "Incompleteness" unfolds its surprisingly accessible story with dignity, tenderness and awe.

News of Gödel's Platonism, or Einstein's, might surprise readers familiar with popular interpretations of their work. For centuries, science seemed to be tidying the mess of the real world into an eternal order beautiful and pure -- a heavenly file cabinet labeled mathematics. Then, in the early 20th century, Einstein published his relativity theory, Werner Heisenberg his uncertainty principle and Gödel his incompleteness theorem. Many thinkers -- from the logical positivists with whom Gödel drank coffee in the Viennese cafes of the 1920's to existentialists, postmodernists and annoying people at cocktail parties -- have taken those three results as proof that reality is subjective and we can't see beyond our noses. You can hardly blame them. As Goldstein points out, the very names of the theories seem to mock the notion of objective truth. But she makes a persuasive case that Gödel and Einstein understood their work to prove the opposite: there is something greater than our little minds; reality exists, whether or not we can ever touch it.

It's appropriate, though sad for Gödel, that his work has been interpreted to have simultaneously opposite meanings. The proofs of his famous theorems rely on just that sort of twisty thinking: statements like the famous Liar's Paradox, "This statement is false," which flip their meanings back and forth. In the case of the Liar's Paradox, if the statement is true, then it's false -- but if it's false, then it's true. Like that paradox, an assertion that talks about itself, Gödel's theorems are meta-statements, which speak about themselves. Because Gödel made so much of self-reference and paradox, previous books about his work -- like Douglas Hofstadter's "Gödel, Escher, Bach" -- tend to emphasize the playfulness of his ideas. Not Goldstein's. She tells his story in a minor key, following Gödel into the paranoia that overtook him after Einstein's death, growing out of his loneliness and unrelenting rationality. After all, paranoia, like math, makes people dig deeper and deeper to find meaning.

Gödel's work addresses the core of mathematics: finding proofs. Proofs are mathematicians' road to truth. To find them, mathematicians from the ancient Greeks on have set up systems consisting of three basic elements: axioms, true statements so intuitively obvious they are self-evident; rules of inference, logical principles indicating how to use axioms to prove new, less obviously true statements; and those new true statements, called theorems. (Many Americans met axioms and proofs for the first and last time in 10th-grade geometry.) A century ago, mathematicians began taking these systems to an extreme. Since mathematical intuition can be as unreliable as other kinds of intuitions -- often things that seem obvious turn out to be just plain wrong -- they tried to eliminate it from their axioms. They built new systems of arbitrary symbols and formal rules for manipulating them. Of course, they chose those particular symbols and rules because of their resemblance to mathematical systems we care about (such as arithmetic). But, by choosing rules and symbols that work whether or not there's any meaning behind them, the mathematicians kept the potential corruption of intuition at bay. The dream of these formalists was that their systems contained a proof for every true statement. Then all mathematics would unfurl from the arbitrary symbols, without any need to appeal to an external mathematical truth accessible only to our often faulty intuition.

Gödel proved exactly the opposite, however. He showed that in any formal system complicated enough to describe the numbers and operations of arithmetic, as long as the axioms don't lead to contradictions there will always be some statement that is not provable -- and the contradiction of it will not be provable either. He also showed that there's no way to prove from within the system that the system itself won't give rise to contradictions. So, any formal system worth bothering with will either sprout contradictions -- which is bad news, since once you have a contradiction, you can prove anything at all, including 2 + 2 = 5 -- or there will be perfectly ordinary statements that may well be true but can never be proved.

You can see why this result rocked mathematics. You can also see why positivists, existentialists and postmodernists had a field day with it, particularly since, once you find one of those unprovable statements, you're free to add it to your system as an axiom, or else to add its complete opposite. Either way, you'll get a new system that works fine. That makes math sound pretty subjective, doesn't it?

Well, Gödel didn't think so, and his reason grows beautifully from his spectacular proof itself, which Goldstein describes with lucid discipline. Though the proof relies on a meticulous, fiddly mechanism that took an entire semester to build up when I studied logic as a math major in college, its essence fits magically into a few pages of a book for laypeople. It can even, arguably, fit in a single paragraph of a book review -- though that may be stretching.

To put it roughly, Gödel proved his theorem by taking the Liar's Paradox, that steed of mystery and contradiction, and harnessing it to his argument. He expressed his theorem and proof in mathematical formulas, of course, but the idea behind it is relatively simple. He built a representative system, and within it he constructed a proposition that essentially said, "This statement is not provable within this system." If he could prove that that was true, he figured, he would have found a statement that was true but not provable within the system, thus proving his theorem. His trick was to consider the statement's exact opposite, which says, "That first statement -- the one that boasted about not being provable within the system -- is lying; it really is provable." Well, is that true? Here's where the Liar's Paradox shows its paces. If the second statement is true, then the first one is provable -- and anything provable must be true. But remember what that statement said in the first place: that it can't be proved. It's true, and it's also false -- impossible! That's a contradiction, which means Gödel's initial assumption -- that the proposition was provable -- is wrong. Therefore, he found a true statement that can't be proved within the formal system.

Thus Gödel showed not only that any consistent formal system complicated enough to describe the rules of grade-school arithmetic would have an unprovable statement, but that it would have an unprovable statement that was nonetheless true. Truth, he concluded, exists "out yonder" (as Einstein liked to put it), even if we can never put a finger on it.

John von Neumann, the father of game theory, took up Gödel's cause in America; in England, Alan Turing provided an alternative proof of Gödel's theorem while inventing theoretical computer science. Whatever Gödel's work had to say about reality, it changed the course of mathematics forever.

Polly Shulman is a contributing editor for Science magazine and has written about mathematics for many other publications.

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